Computing and Informatics, Vol. 28, 2009, 1001–1013, V 2009-Mar-2

PLATEAU PROBLEM IN THE WATERSHED TRANSFORM

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Revised manuscript received 9 December 2008

Abstract. The watershed transform is one of best known and widely used methods for image segmentation in mathematical morphology. Since the definition, deriving from geology and nature observation is quite intuitive and straightforward to implement; many fast and powerful algorithms for watershed transform have already been presented. However, there still occur problems when one wishes to achieve a precise solution on blurred or noised image. The same range of problems is faced when a plateau occurs in the image. In this paper several methods for plateau reduction are discussed and some novel ideas proposed. All algorithms are performed on a set of both natural and synthetic images.

Keywords: Mathematical morphology, watershed transform, watershed definition, plateau reduction

Mathematics Subject Classification 2000: 68U10 – Image Processing

1 INTRODUCTION

The watershed transform is the basic morphological tool for image segmentation [3, 12, 16, 11, 4]. Watershed lines, also called divide lines, are a topographical concept. Since mathematical morphology considers grey-scale image to be a set of points in three-dimensional space, the third dimension being the grey level, light and dark areas of the image may be interpreted as hills and hollows in a landscape [2, 14]. A drop of water falling on a topographical surface follows the steepest descent path until it stops when reaching a regional minimum. The divide lines of the domains of attraction of rain falling over the region are called watershed lines (Figure 1).

The watershed transform can be classified as a region-based segmentation approach and is a process of isolating objects in the image from the background, i.e., partitioning the image into disjoint regions homogeneous with respect to some property, such as grey value or texture. This approach has been successfully applied to many image segmentation problems, like white matter/grey matter segmentation in MR images [5, 11] mammogram analysis [10, 11], solar image analysis [9, 11], aerial and satellite image recognition [7]. Since the first definitions by Digabel and Lantuéjoul, later improved by Beucher and Lantuéjoul [13] many fast and effective algorithms have been presented. Apart from others, specially dedicated for parallel or hardware implementation [12], two classes of sequential algorithms play a significant role in the watershed transform. One is based on the specification of a recursive algorithm by Vincent and Soille [16], the other on distance functions by Meyer [8]. The latter will be a point of further discussion in the following part of this paper.

Although intensively explored in the last few years, watershed segmentation still faces problematic issues, like oversegmentation or thick or misplaced watershed lines, especially when applied to blurred or heavily noised image [5]. Preprocessing is thus an important operation to be done before the watershed transform itself. Filtering by morphological closing or opening is a widely used method for image denoisng [15]. However, it may not lead to a sufficient results when the structure of the image causes that important information is lost by performing any shape-reducing operation. Alternative proposals are markers reducing a number of regional minima [7, 6, 11] or viscous flooding simulation [15]. All of them should lead to a better performance of image segmentation and produce smoother and better distributed segments.

In this paper another anomaly will be discussed, which is a *plateau problem* [12]. The trajectory of a drop of water is disturbed if the relief is not smooth, i.e. the steepest descending path is not explicitly defined. This happens for example when the path comes across a flat area – a set of neighbouring pixels of equal greyscale value. There exist algorithms for reducing such a plateau, usually based on ascending its internal part [12, 1, 3]. A new concept, in which alternatively the external edges are ascended, will be discussed against the traditional concept in the following chapters.

2 MEYER WATERSHED DEFINITION

Discussion in the following section will be based on Meyer watershed definition according to a topographical distance [8, 12].

2.1 Geodesic Distance (Discrete Case)

Let $A \subseteq \mathbb{Z}^n$, $a, b \in A$. The *geodesic distance* $d_A(a, b)$ between a and b within A is the minimum path length among all paths within A from a to b. Let B be a subset of A. Then $d_A(a, B) = \min_{b \in B}(d_A(a, b))$. Assume $B \subseteq A$ being partitioned into k connected components B_i, $i = 1, \ldots, k$. The *geodesic influence zone* of the set B_i within A is defined as

$$iz_A(B_i) = \{p \in A : \bigwedge_{j \in [1,\ldots,k] \setminus \{i\}} d_A(p, B_i) < d_A(p, B_j)\}. \tag{1}$$

Let the union of the geodesic influence zones of all connected components $B_i \subseteq B$ be defined as follows:

$$IZ_A(B) = \bigcup_{i=1}^{k} iz_A(B_i). \tag{2}$$

The complement of the set $IZ_A(B)$ within A is called a *skeleton by influence zones* (SKIZ)

$$SKIZ_A(B) = A \setminus IZ_A(B). \tag{3}$$

2.2 Definition of a Watershed Transform by Topographical Distance

Generalisation of the introduced above skeleton by influence zones (SKIZ) may lead to the definition of a watershed transform on grey-scale images. We follow the definition given by Meyer in [8]. A *digital grid* $G = (V, E)$ is a special graph defined on the domain of image pixels as vertices (V) and a subset of $V \times V$ defining the connectivity as edges (E) (usually in 4- or 8-connectivity). Let f be a digital *lower complete* grey-scale image. Lower completeness means that each non-minimum pixel has a neighbour of lower grey value. This guaranties no plateau occurring in the image (the process of lower completion will be a subject of the next section).

Given a point p we define a *lower slope* of f at the point as

$$LS(p) = \max_{q \in N_G(p) \cup \{p\}} \left(\frac{f(p) - f(q)}{d(p, q)} \right), \tag{4}$$

where $N_G(p)$ is the set of neighbours of pixel p on the grid $G = (V, E)$, and $d(p, q)$ is the distance associated to the edge (p, q) (for $q = p$ the expression is defined to be zero).

The set of lower neighbours q of p, for which the slope $(f(p) - f(q))/d(p, q)$ is maximal, i.e. equals the value $LS(p)$, is denoted by $\Gamma(p)$. The set of pixels q, for which $p \in \Gamma(q)$, is denoted by $\Gamma^{-1}(q)$.

We call $(p_0, p_1, \ldots, p_n)$ a *path of steepest descent* from $p_0 = p$ to $p_n = q$ if $p_{i+1} \in \Gamma(p_{i+1})$ for each $i = 0, \ldots, n-1$. A pixel q belongs to the *downstream* of p, if there exists a path of steepest descent from p to q. A pixel q belongs to the *upstream* of p, if p belongs to the downstream of q.

The *watershed* is a set of all points p which are in the upstream of at least two regional minima, i.e. there are at least two paths of steepest descent starting from p which lead to different minima. Comparing with the previous definition of the skeleton by influence zones (SKIZ), watershed would be the complement of a sum of all catchment basins (influence zones) related to the regional minima of the image.

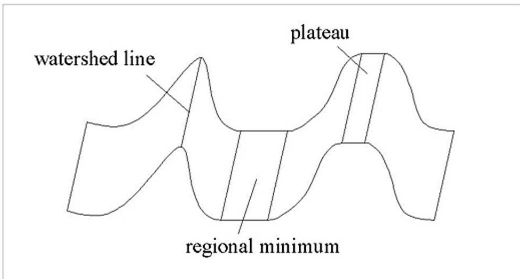


Fig. 1. Topographical relief

3 PLATEAU REDUCTION

3.1 Definition of the Problem

The example of an algorithm based on the above Meyer watershed definition may be given as follows [12]: the paths between points of a catchment basin and the corresponding minimum are paths of steepest descent. Let the relation be inverted. Let all regional minima be labelled with distinct labels. Starting from the boundary pixels of the minima, label all pixels q in the set $\Gamma^{-1}(q)$ of all steepest upper neighbours of the current pixel p by the label of p, unless q is already labelled and the label is different from that of p. In that case label q as a watershed pixel.

However, as indicated in the definition, the initial image has to be lower-complete, i.e. no non-minimum plateau is allowed. Hence, an additional preprocessing step is always required before a performance of the watershed transform itself.

3.2 Basic Plateau Reduction Algorithm

An additional ordering of the plateau pixels is required. The usual solution is to compute the geodesic distance of each plateau pixel from the lower border of the plateau. Then, the internal part of the plateau is being gradually ascended [12, 3, 1].

Let $d(p)$ be the length of the shortest path from a pixel p to any pixel q with a grey value lower than that of p. If p belongs to the image regional minimum, $d(p)$ is defined to be 0. Let then L_c be the maximum of all values $d(p)$ calculated for every pixel p belonging to the image domain. The formal definition of the *lower completion* f_{LC} of the image f should be given as follows:

$$f_{LC}(p) = \begin{cases} L_c \cdot f(p) & \text{if } d(p) = 0 \\ L_c \cdot f(p) + d(p) - 1 & \text{otherwise.} \end{cases} \quad (5)$$

An algorithm for lower completion using a FIFO queue based on the above definition is given in [12]. As seen on a diagram below (Figure 2) the algorithm naturally smooths the relief reducing all non-minima plateaus.

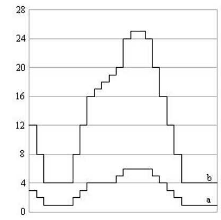


Fig. 2. Lower-completion: a) original image, b) lower complete

This is the situation that may really happen in the nature. A stream of water, when meeting a plateau, doesn't stop but under the pressure finds a way to finally fall down on a lower area. The watershed algorithm performed on the new lower-complete image would simulate this process. However, this doesn't need to be the result we really expect in the image segmentation.

Given an image as in (Figure 3) one may expect to detect the borders surrounding a top plateau in the middle part. Instead, applying the standard lower-completion method we obtain a regular cone based on the whole image domain and as a result of segmentation a smooth surface with no watershed line.

3.3 Watershed on a Gradient Image

It is a common practice in image segmentation to apply the watershed transform to a *morphological gradient* of the image [11]. Several methods for gradient computing are to be found in literature, the basic being defined as a difference between the original image and its morphological erosion [11, 2]. Let f be the original image. Then

$$grad_B(f) = f - \epsilon_B(f), \quad (6)$$

where B is a structuring element (usually a flat 3×3 square), and ϵ is the erosion symbol.

As an operation reflecting differences between neighbouring pixel values, the morphological gradient is often performed in a contour detecting process. When applied as a preprocessing operation before the watershed transform, gradient usually visibly improves the result.

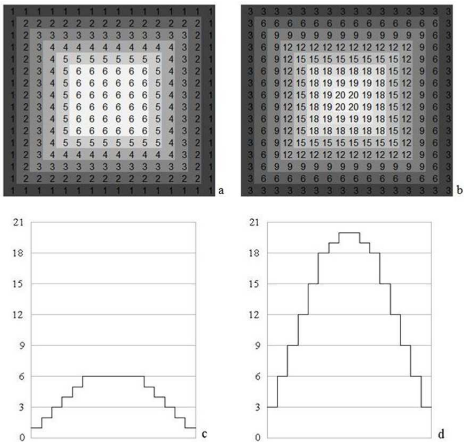


Fig. 3. Top-plateau reduction: a), c) original image and its cross-section, b) d) image and cross-section after the traditional lower-completion transform

In a case of the example in (Figure 3), gradient achieves higher value on the ascending parts of the image, while on the central plateau and external boundary it is equal to 0 (Figure 4).

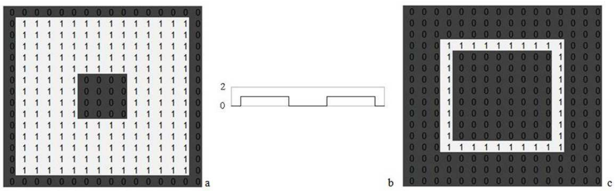


Fig. 4. Image gradient a), b) cross-section of the gradient, c) watershed line

A new top-plateau occurs on the gradient relief placed in the ascending area of the original – that happens because the slope in the example is constant. As a result we need to lower-complete the gradient before applying watershed on it. The interior of the plateau will be slightly ascended and finally the watershed line will be placed exactly in the middle of the gradient plateau (Figure 4 a)). The final result is much better than when computing the watershed directly on the given image; however, the divide line still doesn't fit exactly a border of the initial top plateau. In the next section a new solution will be discussed.

3.4 The Proposed Method – Ascending Edges of the Plateau

Assuming plateau occurring in an image being a significant part for the segmentation process, one may expect its borders to be precisely detected. Hence, the idea of this chapter is to propose a novel solution, where the plateau borders are ascended before any other preprocessing operations is done on the image.

First all plateau areas have to be detected. This may be done by generating an image-size mask, where all plateau pixels are given value 1, while the others will be of value 0. The formal definition of the plateau should be given as follows:

$$pla_{conn}(p) = \begin{cases} 0 & \text{if exists } q \in conn(p) : f(q) < f(p) \\ 1 & \text{otherwise,} \end{cases} \tag{7}$$

where $conn(p)$ denotes a set of neighbouring pixels q of p defined in a connectivity grid – usually 4- or 8-connectivity grid is used.

Two operations may be done before the actual plateau reduction. First, as they do not disturb the further labelling process, all the regional minima will be excluded from the set of plateau pixels by simply subtracting regional minima mask from the plateau mask. Then all pixels neighbouring the plateau and of the same value as the plateau will also be given value 1 on the plateau mask. This is done for a better detection of the plateau edges.

The plateau edges are generated by a gradient-based formula:

$$edge_B(pla) = pla - \epsilon_B(pla), \tag{8}$$

where B is a structuring element for the erosion.

The lower-completion operation will be based on a following formula:

$$f(p) = 2 * f(p) + edge(p). \tag{9}$$

As a result of the defined operation we obtain a lower-complete relief, where all external plateau borders are emphasised and plateau interiors naturally turned into regional minima (Figure 5).

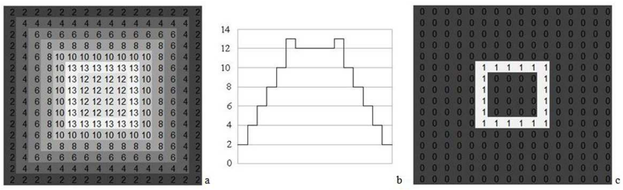
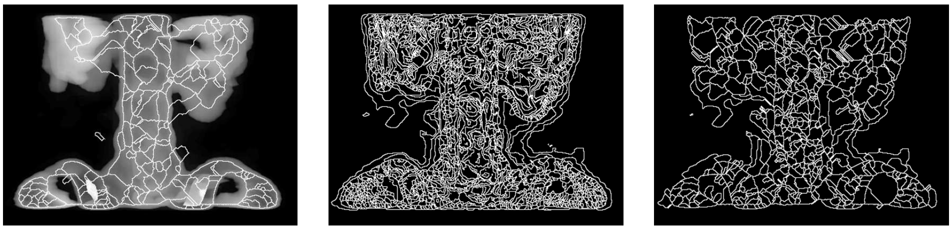


Fig. 5. Lower completion by the proposed method a), b) cross-section, c) watershed line

In Figure 5 c) we can see the watershed line placed exactly on the border of the top plateau. As seen in the given example the proposed method gives better results than any other discussed, when taking into consideration detection of the plateau borders. Although the example was an artificial image with a specially designed relief, the next section will ensure that the discussed method gives equally good results when applied to natural images. Moreover, in most cases it is performed faster than the traditional lower-completion method.

4 DISCUSSION OF THE RESULTS

In the following examples differences concerning results of the watershed transform based on various lower-completion methods are shown. It is possible to notice some change of the results, mainly on blurred areas. In the initial example a medical image is being processed.



a) b) c)

Fig. 6. Watershed based on various lower-completion algorithms. a) basic algorithm, result superimposed to the original image, b) gradient method, c) the proposed algorithm

In Figure 6 a) the basic lower-completion algorithm was performed. As expected, although the segmentation seems good, not all of the plateau borders are seen. The result is changed when applying watershed to the gradient image (Figure 6 b)). The weak point, however, is that too many watershed lines are generated lowering the clearance of the result image. The proposed method (Figure 6 c)) detects significant plateau borders lowering the strong oversegmentation effect observed in the gradient method.

4.1 Evaluation Criteria

For a better explanation of the characteristics of the new method, two evaluation criteria were defined to compare with other algorithms:

1. ability of bright shapes detection,
2. accuracy of the watershed line.

As declared in previous chapters, the proposed method does not concern the watershed algorithm itself, but the lower completion process, which is a necessary, preprocessing step reducing image plateaus. Hence, the evaluation of the lower completion method is done by the results of a standard watershed segmentation applied to a specifically lower completed image.

The following example is a synthetic image designed to show how equal greyscale value bright shape of blurred edges may be interpreted in the segmentation process. A horizontal cross-section of the image is as follows:

$$cs = [\underbrace{240, 240, \ldots, 240}_{60 \quad pixels}, 239, 238, \ldots, 5, 4, 3, 2, 1, 0].$$

Fig. 7. Synthetic image: the left part is a 60-pixel long plateau

One can observe a bright constant value plateau on the left and a constantly lowering, smooth slope falling to the right edge – a regional minimum of the image. The basic lower completion computed as given by (5) consequently arises the left side of the plateau, thus forming a slope falling to the right side (see also Figure 2). Hence, no dividing line is produced by the watershed algorithm.

A cross-section of the image gradient (Equation (6)) is of value 0 along the plateau and of value 1 along the slope:

$$gradient = [\underbrace{0, 0, \ldots, 0}_{59}, 1, 1, \ldots, 1, 0].$$

The gradient transformation itself produces a new plateau of value "1". The basic lower completion applied on the gradient arises the internal part of the new plateau, thus producing two slopes, falling to the left and right edges of the plateau. The watershed transform produces then a dividing line in the middle of the gradient plateau, that is in the middle of the slope of the original image (Figure 8 a)).

$$watershed = [0, 0, \ldots, 0, 1, 0, \ldots, 0]$$

The proposed method of lower completion focuses on arising all edges of the plateau found in the given image, thus transforming the plateau into image regional minima. Hence, as computed by Equation (9), the cross-section after lower completion would be as follows:

$$lowercomplete = [\underbrace{480, 480, \ldots, 480}_{59}, \mathbf{481}, 478, 476, \ldots, 4, 2, 0].$$

Then the watershed line will appear exactly on the last pixel column of the plateau (Figure 8 b)).

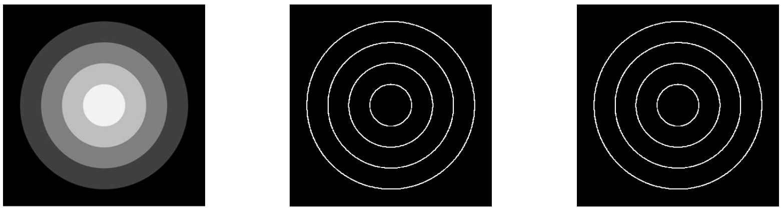
$$watershed = [0, 0, \ldots, 0, \underbrace{1}_{position\quad no.60}, 0, \ldots, 0]$$

a)

b)

Fig. 8. Watershed segmentation: a) basic lower completion on image gradient, b) the proposed method

The second example shows how the mentioned methods of lower completion affect the watershed segmentation results on sharp edges. The initial image is another synthetic one composed of concentric rings of constant grey-scale value, with the brightest circle inside. As might be computed, the basic lower completion causes that again no watershed line is produced. Instead, both gradient-based and the proposed method give the same, precise results.



a) b) c)

Fig. 9. Watershed segmentation: a) original image, b) basic lower completion on image gradient, c) the proposed method

The third example is a natural image, a black and white solar photography. Again, all the three methods of lower completion are applied before the final watershed segmentation and three different results can be observed.

All of the methods give a precise dividing line on the edge of the dark shape covering the sun. Much difference appears as well around the shape as inside it. The basic (Figure 10 b)) and the new (Figure 10 d)) methods produce visually similar lines, though in the latter the division is more dense. In many cases, the gradient-based algorithm (Figure 10 c)) gives different contours. The area of particular interest in case of discussing the use of the proposed algorithm is the upper-left shape, surrounded by a white frame in (Figure 10 a)). Analysis of the pixel value matrix allows to find a brighter shape of a constant value inside the box. Both gradient and new algorithm detect it correctly, while in the basic method it is missing.

Characteristics of the three analysed methods of lower completion according to the defined criteria are collected in the table below (Table 1). The selected criteria emphasise the new algorithm's ability to detect bright, constant value shapes in the image.

Since lower completion is just one step in the segmentation process, it is also influenced by other operations, like sharpening or denoising. Thus, a choice of particular methods as well as the order of applying must always be the subject of a careful study.

5 SUMMARY

As shown by the examples the choice of a plateau reduction method has a significant influence on the watershed transform. Depending of the expected results a proper algorithm should be chosen. The proposed lower completion method gives some

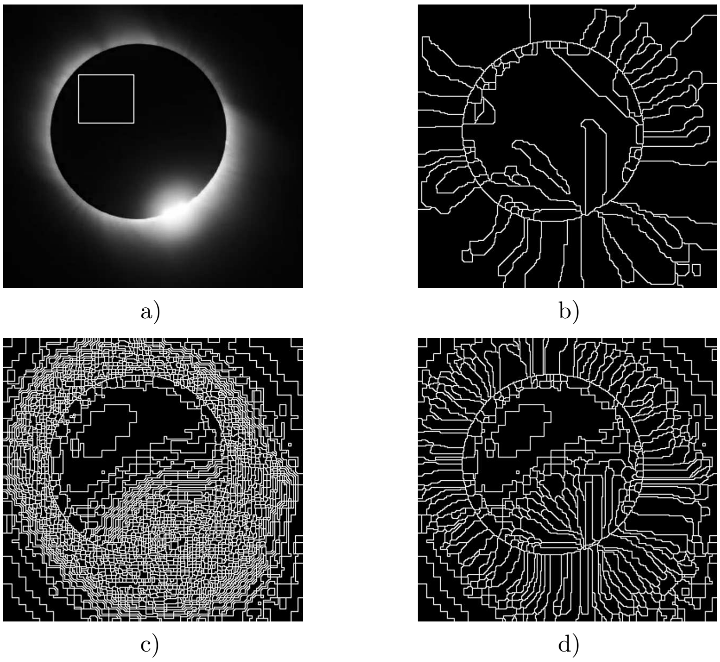


Fig. 10. Watershed segmentation: a) original image, b) basic lower completion, c) basic lower completion on gradient, d) the proposed method

		basic lower completion	gradient based method	new method
bright shape detection		no	yes	yes
accuracy of the dividing line	on sharp edges	-	good	good
	on blurred edges	-	offset/oversegmentation	good

Table 1. Characteristics of chosen methods of plateau reduction according to the defined criteria

alternative idea which in many cases would lead to better performance of the segmentation process. Particular cases where the best results were obtained are those, where brighter shapes are surrounded by a blurred background.

Further research may also concern improvement an testing of the proposed algorithm on various sets of images, as implementation to other research areas, where clustering methods are used.

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